

9. N. A. Zheltukhin, "The determinant method of solving the Orr-Sommerfeld equation," in: Aerodynamics and gasdynamics [in Russian], Nauka, Novosibirsk (1973).
10. V. A. Sapozhnikov, "Solution of the eigenvalue problem for ordinary differential equations by the trial-run method," in: Proceedings of All-Union Seminar on Numerical Methods of the Mechanics of a Viscous Fluid (II) [in Russian], Nauka, Novosibirsk (1969).
11. V. A. Sapozhnikov and V. N. Shtern, "Numerical analysis of the stability of plane Poiseuille flow," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1969).

VISCOSITY OF A DILUTE SUSPENSION OF RIGID SPHERICAL PARTICLES  
IN A NON-NEWTONIAN FLUID

Yu. I. Shmakov and L. M. Shmakova

UDC 532.135

Consider the perturbations introduced by a rigid spherical particle of radius  $a$  suspended in a non-Newtonian fluid flow having a parallel velocity gradient

$$v_x = -(q/2)x, v_y = -(q/2)y, v_z = qz \quad (1)$$

and satisfying the Ostwald-Deville law

$$P = -pE + m(I/2)^{(n-1)/2} S, \quad (2)$$

where  $v_x, v_y, v_z$  are the velocity components in a Cartesian coordinate system  $Oxyz$  with origin at the center of the particle;  $q$  is the constant;  $P$  is the stress tensor;  $\dot{S}$  is the strain rate tensor with components  $\dot{S}_{ij} = \partial v_i / \partial x_j + \partial v_j / \partial x_i$ ,  $i, j = 1, 2, 3$ ;  $I$  is the second invariant of the tensor  $\dot{S}$ ;  $p$  is the pressure,  $E$  is the unit tensor,  $m$  is the consistency index; and  $n$  is the index of non-Newtonian behavior.

Transforming to a spherical coordinate system  $(r, \theta, \varphi)$ , we introduce the stream function  $\psi(r, \theta)$ , which is related to the velocity components by the expressions

$$v_r = -\frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}. \quad (3)$$

Now the equations of motion for a power-law fluid are written as follows, neglecting inertial forces (the generalized Reynolds number with respect to the particle is small):

$$\begin{aligned} \frac{\partial p}{\partial r} &= m \left( \frac{I}{2} \right)^{\frac{n-1}{2}} \left[ -\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} E^2 \psi + \frac{n-1}{2} \left\{ 2 \frac{\partial v_r}{\partial r} \frac{\partial \ln I}{\partial r} + \frac{1}{r} \left( r \frac{\partial}{\partial r} \frac{v_\theta}{r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right) \frac{\partial \ln I}{\partial \theta} \right\} \right], \\ \frac{1}{r} \frac{\partial p}{\partial \theta} &= m \left( \frac{I}{2} \right)^{\frac{n-1}{2}} \left[ \frac{1}{r \sin \theta} \frac{\partial}{\partial r} E^2 \psi + \frac{n-1}{2} \left\{ \left( r \frac{\partial}{\partial r} \frac{v_\theta}{r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right) \frac{\partial \ln I}{\partial r} + \frac{1}{r^2} \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right) \frac{\partial \ln I}{\partial \theta} \right\} \right], \end{aligned} \quad (4)$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right),$$

and the boundary conditions for the problem assume the form

$$\begin{aligned} v_\theta = v_r = 0 \quad \text{at} \quad r = a; \\ v_r = (qr/2)(2 \cos^2 \theta - \sin^2 \theta), \quad v_\theta = -(3qr/2) \sin \theta \cos \theta \quad \text{as} \quad r \rightarrow \infty. \end{aligned} \quad (5)$$

Let  $(n-1)/2 \ll 1$  (the dispersion medium differs only slightly from a Newtonian fluid). Equations (4) can be linearized in this case. Transforming in (4) and (5) to the dimensionless variables  $\bar{r} = r/a$ ,  $\bar{v}_r = v_r/aq$ ,  $\bar{v}_\theta = v_\theta/aq$ ,  $\bar{p} = p/p_\infty$  ( $p_\infty$  is the freestream pressure),  $\bar{\psi} = \psi/a^3 q$ ,  $\bar{I} = I/3q^2$ , we look for a solution of problem (4)-(5) in the form of asymptotic expansions in powers of the small parameter  $\varepsilon = (n-1)/2$ :

$$\begin{aligned} \bar{\psi} &= \bar{\psi}_0 + \varepsilon \bar{\psi}_1 + \varepsilon^2 \bar{\psi}_2 + \dots, \\ \bar{p} &= \bar{p}_0 + \varepsilon \bar{p}_1 + \varepsilon^2 \bar{p}_2 + \dots, \end{aligned}$$

$$\begin{aligned}\bar{v}_r &= \bar{v}_r^{(0)} + \varepsilon \bar{v}_r^{(1)} + \varepsilon^2 \bar{v}_r^{(2)} + \dots, \\ \bar{v}_\theta &= \bar{v}_\theta^{(0)} + \varepsilon \bar{v}_\theta^{(1)} + \varepsilon^2 \bar{v}_\theta^{(2)} + \dots, \\ \bar{I} &= \bar{I}_0 + \varepsilon \bar{I}_1 + \varepsilon^2 \bar{I}_2 + \dots.\end{aligned}$$

In the zeroth approximation the equations of motion take the form

$$\begin{cases} B \frac{\partial \bar{p}_0}{\partial r} = -\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} E^2 \bar{\psi}_0, \\ B \frac{\partial \bar{p}_0}{\partial \theta} = \frac{1}{\sin \theta} \frac{\partial}{\partial r} E^2 \bar{\psi}_0 \end{cases} \quad (6)$$

and differ from the equations of motion for a Newtonian fluid by the presence of the factor  $B = p_\infty / \mu q^{n-1} 3(n-1)/2$ . Eliminating the pressure from (6), we arrive at the following boundary-value problem for the determination of  $\bar{\psi}_0$ :

$$\begin{aligned} E^4 \bar{\psi}_0 &= 0; \quad \bar{\psi}_0 = \partial \bar{\psi}_0 / \partial r = 0 \quad \text{for } \bar{r} = 1; \\ \frac{\partial \bar{\psi}_0}{\partial r} &= -\frac{3\bar{r}^2}{2} \sin^2 \theta \cos \theta, \quad \frac{\partial \bar{\psi}_0}{\partial \theta} = -\frac{\bar{r}^3}{2} (2 \cos^2 \theta - \sin^2 \theta) \sin \theta \quad \text{for } \bar{r} \rightarrow \infty. \end{aligned} \quad (7)$$

In the first approximation we deduce the equations

$$\begin{aligned} B \frac{\partial \bar{p}_1}{\partial r} &= -\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} E^2 \bar{\psi}_1 - \ln \left( \frac{\bar{I}_0}{2} \right) \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} E^2 \bar{\psi}_0 + \\ &+ 2 \frac{\partial \bar{v}_r^{(0)}}{\partial r} \frac{\partial \ln \bar{I}_0}{\partial r} + \frac{1}{r} \left( r \frac{\partial}{\partial r} \frac{\bar{v}_\theta^{(0)}}{r} + \frac{1}{r} \frac{\partial \bar{v}_r^{(0)}}{\partial \theta} \right) \frac{\partial \ln \bar{I}_0}{\partial \theta}, \\ \frac{B}{r} \frac{\partial \bar{p}_1}{\partial \theta} &= \frac{1}{r \sin \theta} \frac{\partial}{\partial r} E^2 \bar{\psi}_1 + \ln \left( \frac{\bar{I}_0}{2} \right) \frac{1}{r \sin \theta} \frac{\partial}{\partial r} E^2 \bar{\psi}_0 + \\ &+ \left( r \frac{\partial}{\partial r} \frac{\bar{v}_\theta^{(0)}}{r} + \frac{1}{r} \frac{\partial \bar{v}_r^{(0)}}{\partial \theta} \right) \frac{\partial \ln \bar{I}_0}{\partial r} + \frac{1}{r^2} \left( \frac{\partial \bar{v}_\theta^{(0)}}{\partial \theta} + \bar{v}_r^{(0)} \right) \frac{\partial \ln \bar{I}_0}{\partial \theta} \end{aligned} \quad (8)$$

along with the boundary conditions

$$\begin{aligned} \bar{\psi}_1 &= \partial \bar{\psi}_1 / \partial r = 0 \quad \text{for } \bar{r} = 1; \quad \partial \bar{\psi}_1 / \partial r < 0 (\bar{r})_z \\ \partial \bar{\psi}_1 / \partial \theta &< 0 (\bar{r}^2), \quad \bar{p}_1 = 0 \quad \text{as } \bar{r} \rightarrow \infty. \end{aligned} \quad (9)$$

The boundary-value problem (7) has a solution corresponding to Newtonian fluid flow with the velocity components (1) past a rigid spherical particle

$$\bar{\psi}_0 = -\frac{\bar{r}^3}{2} \left( 1 - \frac{5}{2\bar{r}^3} + \frac{3}{2\bar{r}^5} \right) \sin^2 \theta \cos \theta. \quad (10)$$

Hence

$$\bar{v}_r^{(0)} = \frac{\bar{r}}{2} \left( 1 - \frac{5}{2\bar{r}^3} + \frac{3}{2\bar{r}^5} \right) (2 \cos^2 \theta - \sin^2 \theta); \quad (11)$$

$$\bar{v}_\theta^{(0)} = -\frac{3\bar{r}}{2} \left( 1 - \frac{1}{\bar{r}^5} \right) \sin \theta \cos \theta; \quad (12)$$

$$\begin{aligned} \bar{I}_0 &= 2 + \frac{5}{r^3} (4 - 18 \sin^2 \theta + 15 \sin^4 \theta) - \\ &- \frac{3}{r^5} (8 - 40 \sin^2 \theta + 35 \sin^4 \theta) + \frac{25}{2} \frac{1}{r^6} (4 - 9 \sin^2 \theta + 6 \sin^4 \theta) - \\ &- \frac{30}{r^8} (4 - 8 \sin^2 \theta + 5 \sin^4 \theta) + \frac{3}{2} \frac{1}{r^{10}} (48 - 80 \sin^2 \theta + 45 \sin^4 \theta). \end{aligned} \quad (13)$$

Integrating (6) under the boundary condition  $\bar{p}_0 = 1$  as  $\bar{r} \rightarrow \infty$ , we find

$$\bar{p}_0 = 1 - \frac{5}{2B} \frac{1}{r^3} (3 \cos^2 \theta - 1). \quad (14)$$

Representing  $\ln(\bar{I}_0/2)$  by an asymptotic expansion in powers of  $1/\bar{r}$ , eliminating the pressure  $\bar{p}_1$  in (8), and making use of (10)-(12), we obtain an equation for the determination of  $\bar{\psi}_1$  in the form

$$E^4 \bar{\psi}_1 = \sum_{s=1}^{\infty} F_s(\bar{r}) \sin^{2s} \theta \cdot \cos \theta, \quad (15)$$

where  $F_s(\bar{r})$  denotes functions known from the solution of the problem in the zeroth approximation, namely polynomials in  $1/\bar{r}$ .

We seek the solution of problem (15), (9) in the form

$$\bar{\psi}_1 = \sum_{s=1}^{\infty} f_s(\bar{r}) \sin^{2s} \theta \cdot \cos \theta. \quad (16)$$

Substituting (16) into (15), we obtain an infinite system of ordinary differential equations for the determination of  $f_s(\bar{r})$ ,

$$\begin{aligned} & \left\{ \frac{d^4}{d\bar{r}^4} - 4s(2s-1) \left[ \frac{1}{\bar{r}^2} \frac{d^2}{d\bar{r}^2} - \frac{2}{\bar{r}^3} \frac{d}{d\bar{r}} - \frac{(s-1)(2s+3)}{\bar{r}^4} \right] \right\} f_s(\bar{r}) + \\ & + 8s(s+1) \left[ \frac{1}{\bar{r}^2} \frac{d^2}{d\bar{r}^2} - \frac{2}{\bar{r}^3} \frac{d}{d\bar{r}} - \frac{2s(2s+3)}{\bar{r}^4} \right] f_{s+1}(\bar{r}) + \\ & + \frac{16s(s+1)^2(s+2)}{\bar{r}^4} f_{s+2}(\bar{r}) = F_s(\bar{r}), \quad s = 1, 2, 3, \dots, \end{aligned} \quad (17)$$

along with the following boundary conditions based on (9):

$$\begin{aligned} f_s(\bar{r}) = df_s(\bar{r})/d\bar{r} = 0 \quad \text{for } \bar{r} = 1; \quad f_s(\bar{r}) < 0(\bar{r}^2), \\ df_s(\bar{r})/d\bar{r} < 0(\bar{r}) \quad \text{as } \bar{r} \rightarrow \infty, \quad s = 1, 2, 3, \dots \end{aligned} \quad (18)$$

We wish to find an approximate solution of problem (15), (9) that will enable us to evaluate the velocity components  $v_r^{(1)}$  and  $v_\theta^{(1)}$  up to terms of order  $O(1/\bar{r}^2)$ . An analysis shows that it is required to solve five equations of the system (17), where the fourth equation does not contain  $f_6(\bar{r})$  and the fifth does not contain  $f_6(\bar{r})$  and  $f_7(\bar{r})$ . These facts make it possible to find  $f_5(\bar{r})$  by solving the fifth equation, and then  $f_4(\bar{r})$ ,  $f_3(\bar{r})$ ,  $f_2(\bar{r})$ , and  $f_1(\bar{r})$  by solving the remaining equations in succession. It is necessary to find  $f_4(\bar{r})$  and  $f_5(\bar{r}) < 0(1/\bar{r}^2)$  in order to determine the integration constants entering into the general solutions of the homogeneous equations for  $s = 1$  and  $2$ . The general solutions of the homogeneous equations of the system (17) have the form

$$\bar{f}_s(\bar{r}) = c_1^{(s)} \bar{r}^{2s+3} + c_2^{(s)} \bar{r}^{2s+1} + c_3^{(s)} \bar{r}^{-2(s-1)} + c_4^{(s)} \bar{r}^{-2s},$$

where  $c_i^s$  ( $i = 1, 2, 3, 4$ ;  $s = 1, 2, 3, \dots$ ) denotes the constant of integration. The particular solutions of the inhomogeneous equations of the system are determined by the form of the functions  $F_s(\bar{r})$ ; the first five of these are

$$\begin{aligned} F_1(\bar{r}) &= 15 \cdot 111/\bar{r}^4 - 15 \cdot 232/\bar{r}^6 - 225 \cdot 167/2\bar{r}^7, \\ F_2(\bar{r}) &= -15 \cdot 483/\bar{r}^4 + 15 \cdot 1039/\bar{r}^6 + 225 \cdot 2061/2\bar{r}^7, \\ F_3(\bar{r}) &= 15 \cdot 855/2 \cdot \bar{r}^4 - 15 \cdot 1869/2\bar{r}^6 - 225 \cdot 3429/\bar{r}^7, \\ F_4(\bar{r}) &= 225 \cdot 4350/\bar{r}^7, \quad F_5(\bar{r}) = -225 \cdot 1875/\bar{r}^7. \end{aligned} \quad (19)$$

The solution of the boundary-value problem (17)-(19) in the given approximation has the form

$$\begin{aligned} f_1 &= A_1 + A_2 \ln \bar{r}; \quad f_2 = A_3; \quad f_3 = A_4 \\ (A_1 &= 5 \cdot 70027/64 \cdot 49 \cdot 33, \quad A_2 = -15/14, \quad A_3 = -309/56, \quad A_4 = 475/112). \end{aligned} \quad (20)$$

Using (20), (16), and (3), we obtain

$$\begin{aligned} \bar{\psi}_1 &= [(A_1 + A_2 \ln \bar{r}) \sin^2 \theta + A_3 \sin^4 \theta + A_4 \sin^6 \theta] \cos \theta; \\ \bar{v}_r^{(1)} &= \frac{A_2}{\bar{r}^2} [\ln \bar{r} (1 - 3 \cos^2 \theta) + 0(1)]; \end{aligned} \quad (21)$$

$$\bar{v}_\theta^{(1)} = \frac{A_2}{2\bar{r}^2} \sin 2\theta. \quad (22)$$

Solving Eqs. (8) in the given approximation, we find the pressure distribution

$$\bar{p}_1 = (2/B\bar{r}^3) [\ln \bar{r} (1 - 3 \cos^2 \theta) + 0(1)]. \quad (23)$$

On the basis of the final solution (11), (12), (14), (21)-(23) we determine the effective viscosity of the suspension, invoking the Einstein energy method [1, 2]. In determining

the dissipation of mechanical energy in the neighborhood of the particle [the solution (21)-(23) being known only at points far from the surface of the particle], we follow a procedure similar to Jeffery's [3], i.e., determine the mechanical energy dissipation in the volume bounded by a spherical surface  $\sigma$  of radius  $R$  in terms of the power of the surface forces applied to that surface:

$$w = \int_{\sigma} (P_{rr}v_r + P_{r\theta}v_{\theta}) d\sigma. \quad (24)$$

Calculating the components  $P_{rr}$  and  $P_{r\theta}$  of the stress tensor (2) and inserting them into (24), we obtain

$$W = w/V_* = m\eta^{n+1}3^{(n+1)/2}[1 + \Phi/2 - \varepsilon\Phi \ln \Phi + O(\varepsilon\Phi)], \quad (25)$$

where  $V_*$  is the volume bounded by the surface  $\sigma$ .

On the other hand, the dissipation of mechanical energy  $W$  can be found in terms of the power of internal forces.

For an incompressible fluid

$$W = P\dot{S}^*/2, \quad (26)$$

where  $\dot{S}^*$  is the strain-rate tensor in the suspension, with components given by the expressions

$$\dot{S}_{xx}^* = \dot{S}_{yy}^* = \frac{2}{V_*} \int_{V_*} \frac{\partial v_x}{\partial x} dV_* = \frac{2}{V_*} \int_{\sigma} v_x \frac{x}{r} d\sigma, \quad (27)$$

$$\dot{S}_{zz}^* = \frac{2}{V_*} \int_{V_*} \frac{\partial v_z}{\partial z} dV_* = \frac{2}{V_*} \int_{\sigma} v_z \frac{z}{r} d\sigma,$$

$$\dot{S}_{ij}^* = 0 \quad \text{for } i \neq j \quad (i, j = 1, 2, 3).$$

If the suspension is dilute (with rigid spheres as the suspended particles) and the dispersion medium differs only slightly from a Newtonian fluid in its rheological properties, we can analyze the suspension in the quasi-Newtonian approximation, i.e., assume that its rheological equation of state has the form

$$P = -pE + \mu_{\text{eff}} \dot{S}^*. \quad (28)$$

Then on the basis of expressions (26)-(28) we obtain an equation for the mechanical energy dissipation

$$W = \mu_{\text{eff}} [1 - 2\Phi - (4/7)\varepsilon\Phi \ln \Phi + O(\varepsilon\Phi)]3q^2. \quad (29)$$

Comparing (29) with (25), we arrive at an expression for the effective viscosity of the investigated suspension

$$\mu_{\text{eff}} = m \left[ 1 + \frac{5}{2}\Phi + \frac{11}{7}\varepsilon\Phi \ln \Phi + O(\varepsilon\Phi) \right] q^{n-1} 3^{\frac{n-1}{2}}. \quad (30)$$

Inasmuch as

$$(1/2)^{(n-1)/2} = 3^{(n-1)/2} q^{n-1}$$

in the given flow, it follows from relation (30) that a dilute suspension of rigid spherical particles in a power-law fluid differing only slightly from a Newtonian fluid behaves as a power-law fluid with an effective consistency index

$$m_{\text{eff}} = m(1 + (5/2)\Phi + (11/7)\varepsilon\Phi \ln \Phi).$$

As  $\Phi \rightarrow 0$  the expression (30) for the effective viscosity of the suspension goes over to the expression for the effective viscosity of the dispersion medium

$$\mu_{\text{eff}} = m(1/2)^{(n-1)/2}$$

and for  $n = 1$  (Newtonian dispersion medium) expression (30) gives the classical result of Einstein.

#### LITERATURE CITED

1. A. Einstein, "Über die von der molekular-kinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen," Ann. Phys., 19 (1906).

2. A. Einstein, "Berichtigung zu meiner Arbeit: 'Eine neue Bestimmung der Molekuldimensionen,'" Ann. Phys., 34 (1911).
3. G. B. Jeffery, "The motion of ellipsoidal particles immersed in a viscous fluid," Proc. R. Soc. London, Ser. A, 102, No. 715 (1922).

FLOW AND HEAT TRANSFER IN THE THERMOGRAVITATIONAL GENERATION MODE

A. F. Polyakov

UDC 532.517.4+536.24

Experimental investigations [1-3] of heat elimination in a turbulent descending fluid flow in vertical heated pipes under conditions of substantial influence of thermogravitation show that the Nusselt number grows monotonically with the increase in the Grashof number for a constant value of the Reynolds number. The analysis performed in [4] for the case of relatively weak influence of thermogravitation showed that the monotonic increase in the Nusselt number in this case is associated with the influence of lift forces on the turbulent transfer under conditions of unstable stratification of the density. The influence of thermogravitation results in an increase in turbulent transfer, in a more filled-out shape of the velocity and temperature profiles, in an increase in the friction drag and heat elimination. The nature of the flow is hence determined for all values of the Grashof number by the influence of the thermogravitational forces on the turbulent transfer. The influence of the thermogravitational forces directly on the average flow (i.e., taking account of the lift forces in the average equation of motion) is substantially less than their influence on turbulence for relatively low values of the Grashof number, and this difference increases more and more with its growth.

The expression in [4] for the coefficient of turbulent momentum transfer  $\epsilon$ ,

$$\frac{\epsilon}{\nu} = \left(\frac{\epsilon}{\nu}\right)_T \left[ 1 + 41 \frac{Gr}{Re_*^4 Pr} \frac{dT^+/d\eta}{Pr_T (dU^+/d\eta)^2} \right]^{1/4}, \quad (1)$$

was used for the case of smallness of the parameter taking account of the lift forces, i.e., when the second member in the square brackets is substantially less than one. Here  $\nu$  is the kinematic coefficient of viscosity;  $(\epsilon/\nu)_T$  is the relative coefficient of turbulent momentum transfer in an isothermal flow;  $Gr = g\beta q_w d^4 / \lambda \nu^2$  is the Grashof number;  $Re_* = v_* d / \nu = Re \sqrt{c_f/2}$ ,  $Re = \bar{u}d/\nu$  is the Reynolds number;  $\beta$  is the coefficient of volume expansion;  $g$  is the acceleration of gravity;  $q_w$  is the thermal-flux density at the wall;  $d$  is the pipe diameter (characteristic dimension);  $\lambda$  is the coefficient of thermal conductivity;  $v_* = \sqrt{\tau_w/\rho}$  is the dynamic velocity;  $\tau_w$  is the tangential friction stress at the wall;  $\rho$  is the density;  $c_f$  is the friction drag coefficient;  $\bar{u}$  is the mean (characteristic) velocity with respect to the transverse section;  $Pr$  is the Prandtl number;  $Pr_T$  is the turbulent Prandtl number;  $T^+ = (\tau_w - t) \cdot \rho c_p v_* / q_w$  is the dimensionless temperature;  $t$  is the temperature;  $\tau_w$  is the wall temperature;  $c_p$  is the specific heat at constant pressure;  $\eta = v_* y / \nu$  is the dimensionless distance to the wall;  $y$  is the distance from the wall along the normal;  $U^+ = u/v_*$  is the dimensionless velocity; and  $u$  is the velocity parallel to the wall in the  $x$  direction.

The second member in the square brackets in (1) characterizes the contribution of thermogravitation to the generation of turbulence as compared with generation because of the average flow, and this member is considered substantially greater than the first term in this paper. This corresponds to the following physical situation when generation of turbulence by the thermogravitational forces is substantially greater than generation by the average flow. We call such a mode the "thermogravitational generation mode." The forced average flow is hence accomplished because of an external circulation source.

Equation (1) is written for the thermogravitational generation mode as

$$\frac{\epsilon}{\nu} = \frac{C}{Pr_T^{1/4}} \frac{Gr^{1/4}}{Re_* Pr^{1/4}} \frac{(dT^+/d\eta)^{1/4}}{(dU^+/d\eta)^{1/2}} \left(\frac{\epsilon}{\nu}\right)_T, \quad (2)$$